Conclusion

For the minimum fuel-fixed range with time either free or specified, the cruise condition based upon the dynamics of Ref. 1 is found not to be a minimizing singular arc by application of the generalized Legendre-Clebsch condition for vector control. This is seen to be consistent with the results for the energy-state approximation for which intermediate values of thrust are not minimizing. Finally, it can be shown that this conclusion holds when the restriction on constant mass is removed.

References

[1]Schultz, R. L. and Zagalsky, N. R., "Aircraft Performance Optimization," *Journal of Aircraft*, Vol. 9, No. 2, Feb. 1972, pp. 108–114.

[2]Bryson, A. E., Jr. and Ho, Y. C., *Applied Optimal Control,* Blaisdell, Waltham, Mass., 1969,

[3]Kelley, H. J., Kopp, R. E., and Moyer, H. G., "Singular Extremals," *Topics in Optimization,* edited by G. Leitmann, Academic Press, New York, 1967,

[4]Jacobson, D. H., "Totally Singular Quadratic Minimization Problems," *IEEE Transactions on Auto. Control,* Vol. AC-16, No. 6, 1971,

[5]Kelley, H. J., "A Second Variation Test for Singular Extremals," *AIAA Journal* Vol. 2, No. 8, Aug. 1964, pp. 1380–1382.

[6]Robbins, H. M., "A Generalized Legendre-Clebsch Condition for the Singular Cases of Optimal Control," Rept. 66-825, 1966, IBM Federal Systems Division Oswego, New York.

[7]Goh, B. S., "The Second Variation for the Singular Bolza Problem," *SIAM Journal of Control,* Vol. 4, 1966, pp. 309–325.

[8]Bryson, A. E., Jr., Desai, M. N., and Hoffman, W. C., "Energy-State Approximation in Performance Optimization of Supersonic Aircraft," *Journal of Aircraft,* Vol. 6, No. 6, Nov.-Dec. 1969, pp. 481–488.

Inviscid Wake-Airfoil Interaction on Multielement High Lift Systems

Alfred Moser* and Carl A. Shollenberger†
McDonnell Douglas Research Laboratories, St. Louis, Mo.

ON multielement airfoils, a boundary layer wake is shed by the main airfoil and flows over the flap elements. A viscous interaction of the wake with the boundary layer of the flap element is almost certain. The question arises: does the wake have an inviscid effect on the flap surface pressure, i.e., would the lift of the flap be affected by the wake even if any viscous or apparent turbulent stresses would be absent? A linearized method for estimating the inviscid wake effect and an attempt to explain the effect are presented, and the results are compared with "exact" numerical calculations using a recently developed singularity method.[1]

According to Küchemann[2] and Thwaites[3] the lift of the flap is affected by the wake in inviscid fluid even for cases where potential flow is present between the wake and the flap. As a qualitative answer, Refs. 2 and 3 mention that the additional lift force on airfoils induced by nonuniformities in the (inviscid) freestream flow always tends to be directed toward the region with the highest velocity or total pressure. In the case of a wake flowing over the upper surface of a flap, the lift of the flap would be reduced by inviscid interaction.

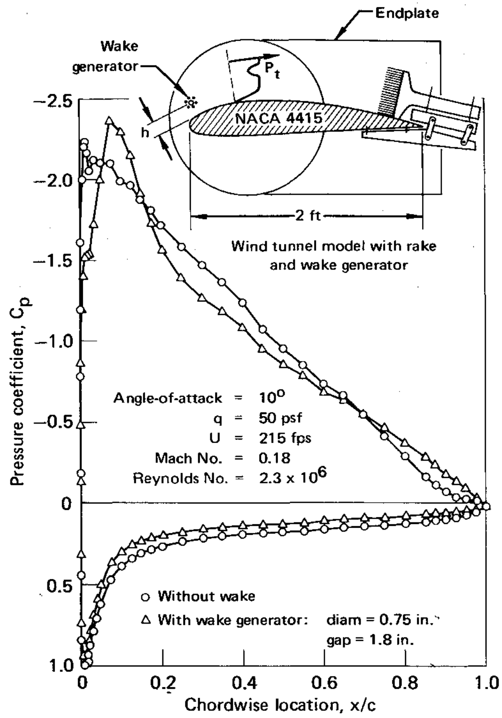

Fig. 1 Experimental airfoil surface pressure distributions with and without wake.[5]

Experimental verification of the inviscid interaction is difficult because viscosity and turbulence effects are always superimposed onto the inviscid effects. Ljungström[4] conducted experiments on a 2-D airfoil model equipped with slat and flap. By different means, he was able to vary the momentum defect of the wake leaving the main wing trailing edge. The measurements of flap surface pressure distributions for different wake momentum defects on otherwise identical configurations provide an indication of the inviscid wake effect. The results show less negative static pressure on the flap upper surface for the larger wake and almost no difference on the flap lower surface, in agreement with the present inviscid calculations. Pressure distributions obtained in recent experiments by the authors[5] are shown in Fig. 1. Here the wake was generated by a bundle of transverse rods near the leading edge of a NACA 4415 section. The difference of the surface pressures measured near midchord with and without the wake is approximately as expected for the inviscid wake interaction; however, other effects of the viscous wake must also be considered.

For further discussion, a simplified flow model is defined as follows: one airfoil (representing the flap) is placed in a two-dimensional, incompressible, inviscid flow which is irrotational except in the region of the wake. The wake is described by total pressure profiles, and it originates at upstream infinity or at a "wake generator" defined below. For this type flow the equations of motion relate the wake total pressure to the local value of the stream function in a unique way for the entire wake. The wake generator in this analysis is a straight line segment in the plane of flow of length, h. The total pressure, H, of fluid crossing this line segment is instantly reduced by ΔH; however, the flow velocity is continuous across the wake generator. The resulting total pressure wake profiles have rectangular shapes. The force on the wake generator, $F = h\Delta H$, is perpendicular to the line segment. The com-

Received September 27, 1973; this research was conducted under the McDonnell Douglas Independent Research and Development Program.

Index category: Aircraft Aerodynamics (Including Components Aerodynamics); Jets, Wakes, and Viscid-Inviscid Flow Interactions; Subsonic and Transonic Flow.

*Research Scientist, Flight Sciences Department; now with Douglas Aircraft Company. Member AIAA.

†Research Scientist, Flight Sciences Department. Member AIAA.

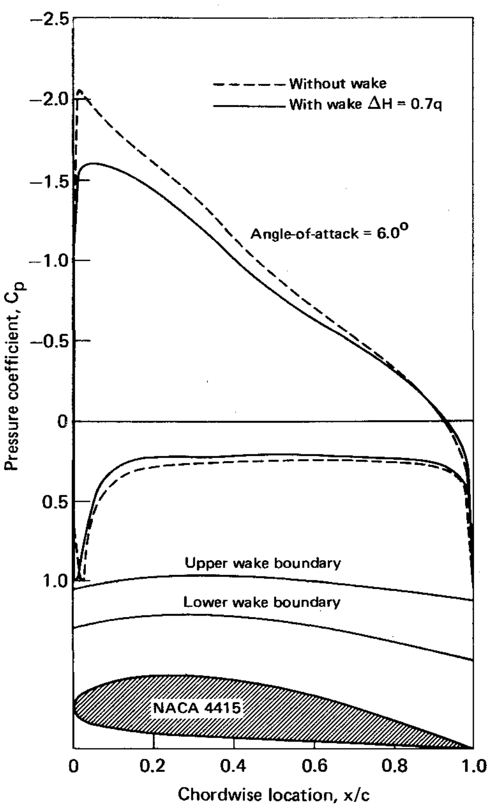

Fig. 2 Inviscid effect of wake on airfoil calculated by singularity technique.

ponent of this force parallel to the freestream direction is called the wake generator drag, D.

The basic problem now consists of determining the change of surface pressures (and forces) on the airfoil induced by the presence of a wake. A related task is to find the location of the wake boundaries. The only way to arrive at a correct answer is to calculate the entire flow field twice, first without the wake and second with a wake. The nonlinearity of the underlying equations of motion precludes superimposing the wake on the potential flow solution. The calculation was conveniently carried out by using the method of Ref. 1 for thick airfoils with the wake handled as a jet with a negative total pressure increment. The results for a sample case roughly corresponding to the tested configuration of Fig. 1 are shown in Fig. 2. The ratio of the inviscid lift loss to the drag, D, on the wake generator was 2.3 in this calculation. This ratio of two perpendicular forces gives some indication of the strength of interaction. The ratio increases for increasing angle-of-attack, for increasing ΔH, and for a decreasing gap between surface and wake. Upon translating this result to an actual two-element airfoil, it is concluded that the inviscid lift loss on the flap may amount to several times the viscous drag of the main airfoil (D corresponds to the viscous drag of the forward element).

A possible explanation of the physics of this interaction follows along with some calculations to illustrate and verify the ideas. The lift of an airfoil is transmitted from the inviscid fluid by static pressures to the body surface. The difference of static pressures on the upper and lower surfaces is produced by downward deflection of stream tubes (or stream layers in two-dimensional flow) in the surrounding flow field. If the fluid between two streamlines is forced to flow along a curved path, the static pressure on the streamline inside the bend is lower than on the outside. This pressure difference, caused by centrifugal forces, is proportional to the product of massflow between the streamlines and mean flow velocity divided by the mean radius of path curvature. In differential form

$$\partial p/\partial n = \rho(v^2/R) \tag{1}$$

This relation is exact for inviscid irrotational and rotational flow with the static pressure, p, the coordinate normal to the streamline, n, the density, ρ, the flow velocity, v, and the radius of streamline curvature, R. The streamlines above and below a lifting airfoil are essentially deflected downward. Integration of Eq. (1) from a point far above the airfoil down to the upper surface indeed results in a negative surface pressure as required for positive lift. Corresponding integration from below the airfoil yields a positive pressure for the lower surface if the airfoil is sufficiently cambered.

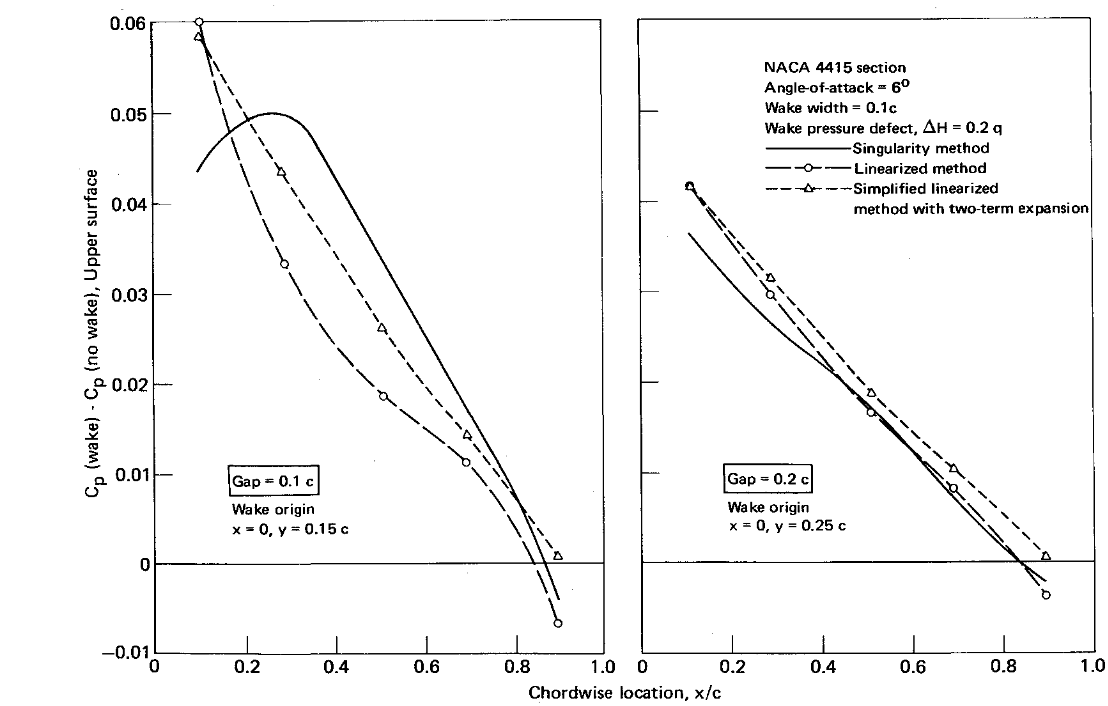

Fig. 3 Inviscid effect of wake on upper surface static pressure calculated by various methods.

An idealized wake is now introduced by reducing the total pressure of the flow between two streamlines which represent the wake boundaries by ΔH. As already mentioned, this change disturbs the entire flow field; however, for the present approximation, it is assumed that the streamline curvature at each point is unaffected. The static pressure differential across the wake is less than across the corresponding stream tube with equal streamline curvature but full freestream total pressure. The flow velocity in the stream tube is now reduced, and a lower static pressure gradient is required to force the flow around the curvature, since the centrifugal force is smaller. Integration of Eq. (1) now results in a less negative surface pressure and thus in a loss of lift. With the definition of total pressure,

$$H = p + 0.5\rho v^2 \tag{2}$$

the change of static pressure at a single point on the airfoil surface is found by integrating Eq. (1) along a equipotential curve originating at the surface point. The distribution of radius of streamline curvature, $R(n)$, along this curve is obtained from a potential flow calculation in the present approximation. The curvature distribution remains undisturbed only for very small values of $\Delta H/q$, where q is the freestream dynamic pressure; therefore, this method is a linearization for small $\Delta H/q$. Wake width and flow angles, however, need not be small. In the present approach, a new function, $f(n)$, is introduced to simplify the integration of the final differential equation

$$f(n) = (C_p(0) - 1)/(C_p(n) - 1) \tag{3}$$

where $C_p(n)$ is the pressure coefficient along the equipotential curve in absence of the wake, and $C_p(0)$ is the value at the surface. The radius function $R(n)$ of the potential flow solution is now expressed by $f(n)$, and the streamline curvature is

$$1/R(n) = 0.5d(\ln f)/dn \tag{4}$$

For a given total pressure wake profile, $H(n)$, and a known curvature function, $f(n)$, a new functional relation, $H(f)$, is determined. The final differential equation is obtained by applying Eqs. (1, 2, and 4) to the flow with and the flow without the wake, by using the assumption of equal curvature distribution in both flows, and by subtracting the resulting equations. Integration of the differential equation is performed from the airfoil surface ($f = 1.0$) to infinity above the airfoil [$f = 1.0 - C_p(0)$], and the change of static surface pressure caused by the presence of the wake becomes

$$\Delta p(0) = \int_{1.0}^{1.0 - C_p(0)} (H(\infty) - H(f))df \tag{5}$$

Figure 3 shows a comparison of the present approximation (dashed curve) with the singularity method of Ref. 1 (solid curve). In this example, the total pressure defect, ΔH, of the wake was only 20% of the freestream dynamic pressure. For the dotted curve, the function $f(n)$ was approximated by a two-term expansion which satisfies the no-wake surface pressure at $n = 0$ and the far field behavior for the given no-wake airfoil circulation. In the present example, the simplified method (dotted curve) seems to be adequate for the linearized problem, except that this curve fails to assume negative values near the trailing edge, as indicated by the solid and dashed curves. This change of sign of the pressure difference is in agreement with the experimental results of Ref. 4.

The good agreement among the three curves in Fig. 3 supports the present qualitative explanation of the inviscid wake effect. It is concluded that an inviscid effect exists and that this effect can be reasonably estimated by the linearized method for the airfoil upper surface and for small total pressure defects. The linearized method is also applicable to weak jets. For stronger wakes, the entire flow field must be calculated by methods such as proposed in Ref. 1 with the wake represented by appropriate singularities.

References

[1]Shollenberger, C. A., "Analysis of the Interaction of Jets and Airfoils in Two Dimensions," *Journal of Aircraft*, Vol. 10, No. 5, May 1973, pp. 267–273.

[2]Küchemann, D., "Comments on Paper 10, Aerodynamics of High-Lift Airfoil Systems," AGARD-CP-102, 1972, pp. 10-25–10-27.

[3]Thwaites, B., "Non-Uniform Mainstreams. The Effects of Variations in the Stream in the Lift Direction," *Incompressible Aerodynamics*, Oxford University Press, Great Britain, 1960, pp. 549–550.

[4]Ljungström, B. L. G., "Boundary Layer Studies on a Two-Dimensional High Lift Wing," FFA Rept. AU-862, Oct. 1972, The Aeronautical Research Institute of Sweden, Bromma, Sweden.

[5]Moser, A., "Wake/Boundary Layer Interaction Wind Tunnel Test," Rept. MDC Q0463, July 1972, McDonnell Douglas Research Laboratories, St. Louis, Mo.